Exterior Space Model for an Acoustic Eigenvalue Problem

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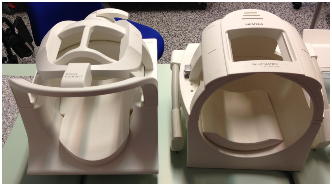
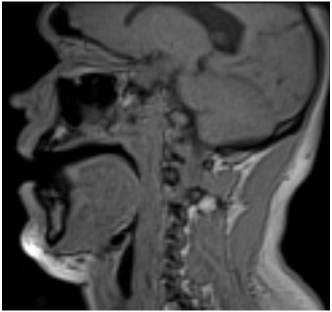
Introduction
Motivation

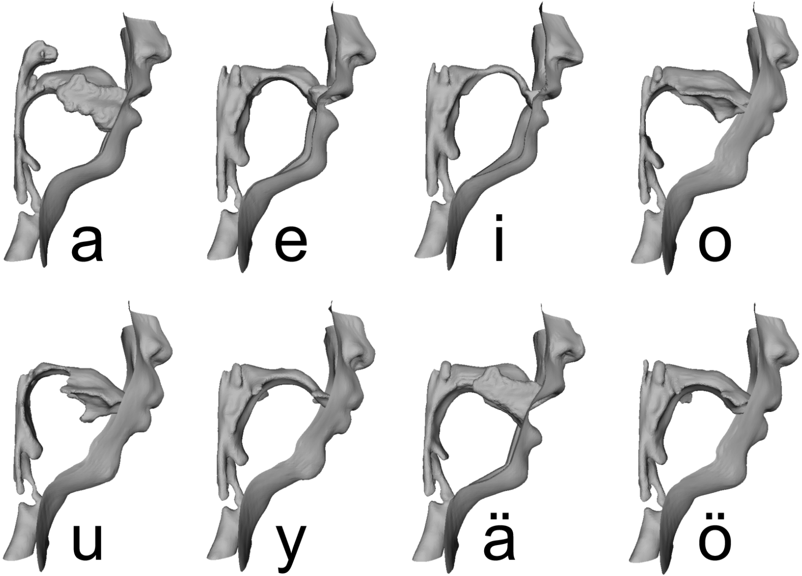
- Long term goal: Modelling human speech production by combining FEM and MRI from the Vocal Tract (VT).
- Applications: Planning surgery that affects speech. Speech synthesis. General phonetics.
- Analysis of vowels: Formants vs. VT resonances – solutions to an eigenvalue problem.
- Assets: 1000+ pairs of VT MRI and simultaneous voice recordings.





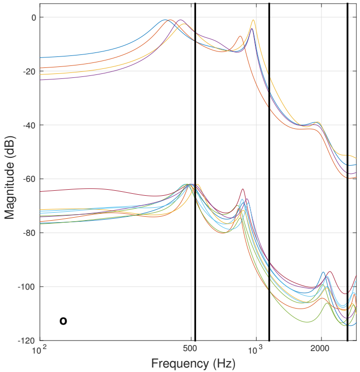
Introduction (2)

Vowels



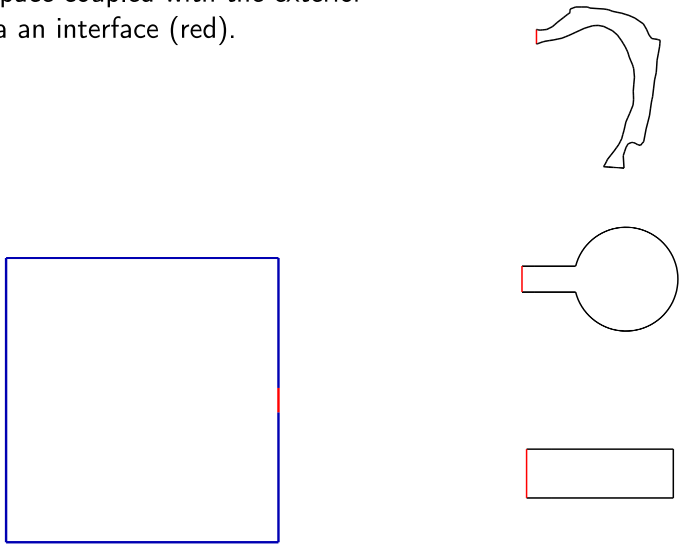
Challenges

- The exterior space affects formant positions.
- Exterior space is large compared to the vocal tract. Unnecessary computational cost.
- Introducing new VT and exterior space geometries. Automation needed for treating large datasets.



Geometries

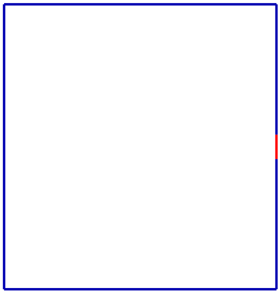
Interior space coupled with the exterior (blue) via an interface (red).

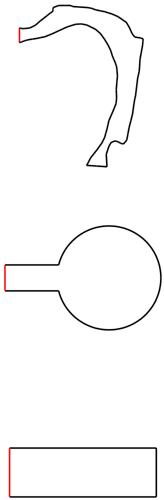


Geometries

Interior space coupled with the exterior (blue) via an interface (red).

The interface is kept fixed so that different interior geometries can be introduced by Nitsche's method.



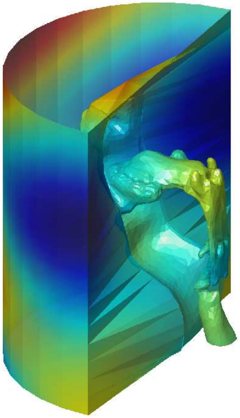


Resonances

- The resonant frequencies are related to the eigenvalue problem: Find $(\lambda, u) \in \mathbb{C} \times V$ such that

$$-c^2 \Delta u = \lambda^2 u,$$

  where V is the solution space.
- Realistic boundary conditions lead to a quadratic eigenvalue problem.

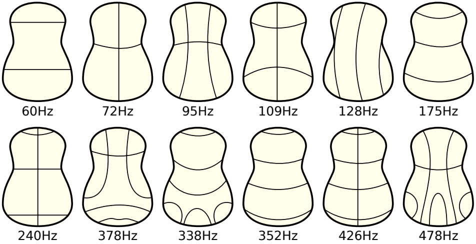


Resonances (2)

- Simplification: $u = 0$ on $\partial\Omega$ (Dirichlet boundary condition)
- Find $(\lambda, u) \in \mathbb{R} \times H_0^1(\Omega)$ s.t.

$$\Delta u = \lambda u.$$

- (Almost) non-physical, but easier to analyse.

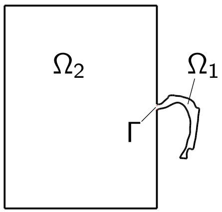


http://commons.wikimedia.org/wiki/File:Chladni_guitar.svg, 10.10.2014

Method

Domain:

$$\begin{aligned} \Omega &:= \Omega_1 \cup \Omega_2 \cup \Gamma \\ \Omega_1 &:= \text{the vocal tract (interior domain)} \\ \Omega_2 &:= \text{the exterior domain} \\ \Gamma &:= \partial\Omega_1 \cap \partial\Omega_2, \text{ the interface} \end{aligned}$$



We can write the discretised problem as

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \mathsf{x}_1 \\ \mathsf{x}_2 \end{bmatrix} = \lambda_0 \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \mathsf{x}_1 \\ \mathsf{x}_2 \end{bmatrix}.$$

Method (2)

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \mathsf{x}_1 \\ \mathsf{x}_2 \end{bmatrix} = \lambda_0 \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \mathsf{x}_1 \\ \mathsf{x}_2 \end{bmatrix}$$

Define

$$B(\lambda) := -(A_{22} - \lambda M_{22})^{-1}(A_{21} - \lambda M_{21}).$$

B is a non-linear function of λ satisfying

$$\mathsf{x}_2 = B(\lambda_0)\mathsf{x}_1.$$

Challenge: approximate the effect of $B(\lambda)$ on an interval $[\lambda_{min}, \lambda_{max}]$ (or several intervals)

Method (3)

$$B(\lambda) = -(A_{22} - \lambda M_{22})^{-1}(A_{21} - \lambda M_{21}).$$

- Low number of degrees-of-freedom on Γ, $\mathrm{range}(A_{21} - \lambda M_{21})$ is contained in the subspace spanned by these *DoF*'s for any λ.
- Oversampling in λ on an interval on every node of Γ:

$$\mathbf{y}_{ij} = (A_{22} - \lambda_i M_{22})^{-1}\mathbf{q}_j.$$

- Use *PCA* to reduce the dimension of the obtained vector space.
- Slight modifications required when using Nitsche's method.

Method (4)

We get the following reduced eigenvalue problem:

$$\begin{bmatrix} A_{11} & A_{12}\widetilde{U} \\ \widetilde{U}^* A_{21} & \widetilde{U}^* A_{22}\widetilde{U} \end{bmatrix} \begin{bmatrix} \mathsf{x}_1 \\ \alpha \end{bmatrix} = \widetilde{\lambda} \begin{bmatrix} M_{11} & M_{12}\widetilde{U} \\ \widetilde{U}^* M_{21} & \widetilde{U}^* M_{22}\widetilde{U} \end{bmatrix} \begin{bmatrix} \mathsf{x}_1 \\ \alpha \end{bmatrix},$$

where x_2 corresponds to $\widetilde{U}\alpha$.

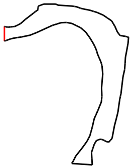
$\widetilde{U}$ does not depend on Ω_1. Computed only once for a given exterior domain.

Results

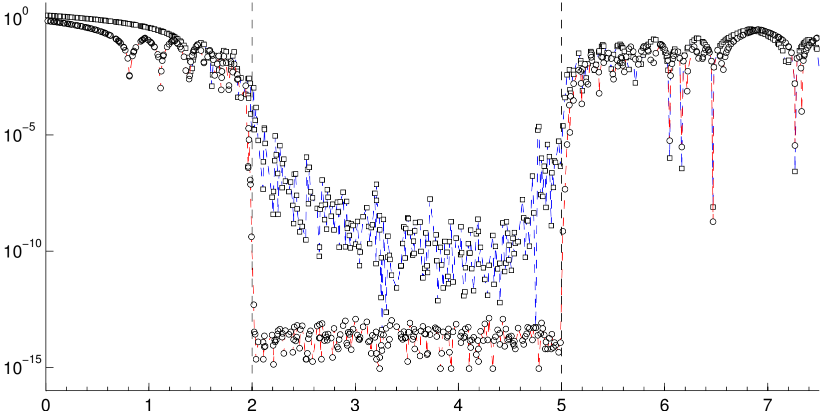
Using the mid-sagittal plane of a vocal tract as Ω_1, we compute

$$\min_i |\lambda - \widetilde{\lambda}_i|,$$

where λ is an eigenvalue of the full problem, and $\widetilde{\lambda}_i$ is an eigenvalue of the reduced problem.



Results

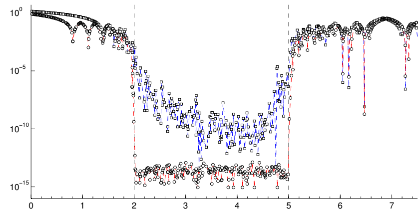


$$\min_i |\lambda - \widetilde{\lambda}_i|$$

Blue: 61 samples. Red: 301 samples.

Results

Ω_2 DoF's	3084
Interval	$[2, 5]$
Samples	301
Reduced dim.	278
λ's on the interval	200

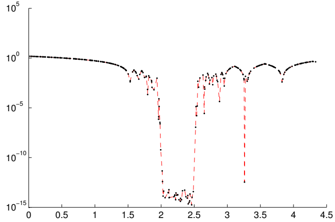


Results

Smaller interval

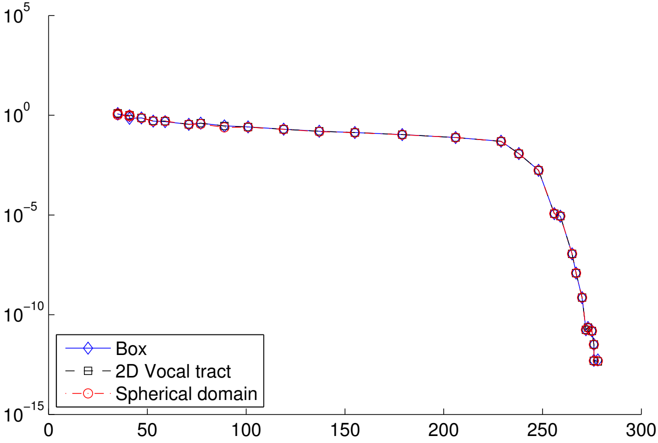
More reduction when using a smaller interval:

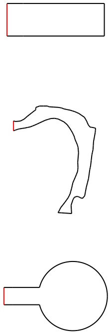
Ω_2 DoF's	3084
Interval	$[2, 2.5]$
Samples	51
Reduced dim.	90
λ's on the interval	34



Results

Convergence





Norm of the error on the interval as a function of the reduced basis dimension.

Improvements & What's next

- Better sampling strategy.
- Choosing a smarter basis on the interface.
- Performing SVD on every node, parallelisation.
- Analysis on convergence.

Thank you

http://speech.math.aalto.fi

Collaborators:
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Department of Oral and Maxillofacial Diseases, Turku University Hospital, and
Medical Imaging Centre of Southwest Finland at Turku University Hospital.