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ON EXTERIOR ACOUSTICS AND SPEECH MODELLING

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Summary. Nitsche's method is applied on an eigenvalue problem motivated by speech modelling, enabling one to use non-matching meshes for the exterior acoustic space and the vocal tract. As a result, resonance computations can be automatically performed on multiple vocal tract geometries.

1 INTRODUCTION

Simulating resonant frequencies in vocal tract geometries extracted from MRI data[1] is an important step in human speech modelling. An ongoing challenge in such simulations is including the exterior acoustic space accurately into the model. The easiest option is to place a Dirichlet boundary condition at the mouth to emulate a constant pressure environment. However, as the MRI machine is a rather constricted space, it most likely has a strong effect on some of the resonances. This has lead to using more detailed models of the exterior space[2].

To validate the results of the resonance simulations, our research group has collected a validation dataset consisting of audio recorded inside an MRI machine simultaneously with the 3D MRI measurements[3]. The validation dataset consists of approximately 1000 different vocal tract geometries. Processing such a large amount of different geometries automatically requires special techniques. The aim of this paper is to present a method that allows one to easily exchange vocal tract and exterior space geometries in the simulation.

Let $\Omega = \Omega_1 \cup \Omega_2$, where the distinct domains Ω_1 and Ω_2 correspond to the vocal tract and the exterior domain of the acoustic space surrounding the head, respectively. See Section 3 for examples of geometries that are actually used. In a typical simulation, Ω_2 remains unchanged while Ω_1 varies between different vowels and patients. The domains Ω_1 and Ω_2 are connected via an interface denoted by $\Gamma = \partial\Omega_1 \cap \partial\Omega_2$. We are interested in the following eigenvalue problem:
Find (λ, u) such that

$$-\Delta u = \lambda u \quad \text{in } \Omega, \ u = 0 \text{ on } \Gamma_D, \ \text{and } \frac{\partial u}{\partial n} = 0 \text{ on } \Gamma_N, \tag{1}$$

where Γ_D and $\Gamma_N = \partial\Omega \setminus \Gamma_D$ denote the Dirichlet and Neumann boundaries, respectively. This problem is a simplification of the actual acoustic problem involving more

complicated boundary conditions. Using the sub-domains defined earlier, the problem can be written as follows:
Find (λ, u_1, u_2) such that

$$\begin{cases} -\Delta\, u_i = \lambda u_i \quad \text{in } \Omega_i,\; u_i = 0 \text{ on } \Gamma_D,\; \text{and } \frac{\partial u_i}{\partial n} = 0 \text{ on } \Gamma_N, \quad i = 1,2, \\ \quad u_1 = u_2 \text{ and } \frac{\partial u_1}{\partial n} = \frac{\partial u_2}{\partial n}, \quad \text{on } \Gamma. \end{cases} \tag{2}$$

In order to easily exchange different interior geometries, we discretise the above system separately with respect to the domains Ω_1 and Ω_2, resulting in triangulations $\mathcal{T}_1$ and $\mathcal{T}_2$, respectively. Nitsche's method is then used to enforce conditions stated in the second equation in (2), i.e, the continuity conditions over the interface Γ. This allows us to use non-matching grids, which considerably simplifies the mesh generation for the domains.

To prepare an extracted vocal tract geometry for use, the space between the edge of the extracted geometry's mouth area and the edge of the spherical surface of the interface is triangulated. Processed geometries can be seen in Figure 2.

2 NITSCHE'S METHOD

In this section, we shortly review Nitsche's method and some aspects related to its implementation. For a detailed mathematical treatment, see Becker et al.[4]. Applying Nitsche's method onto equation (1) leads to the following problem:
Find $(\lambda, u) \in (\mathbb{R}^+, V)$ such that

$$a(u,v) = \lambda b(u,v) \quad \forall v \in V, \tag{3}$$

where

$$V := \left\{ u \in L^2(\Omega) \mid u|_{\Omega_i} \in H^1(\Omega_i), i = 1,2,\; u = 0 \text{ on } \Gamma_D \right\}. \tag{4}$$

The forms $a(\cdot,\cdot)$ and $b(\cdot,\cdot)$ are defined as $b(u,v) = (u,v)_\Omega$, and

$$a(u,v) = \sum_{i=1}^{2} (\nabla u, \nabla v)_{\Omega_i} - \left\langle \left\{ \frac{\partial u}{\partial n} \right\}, [\![v]\!] \right\rangle_\Gamma - \left\langle [\![u]\!], \left\{ \frac{\partial v}{\partial n} \right\} \right\rangle_\Gamma + \alpha \left\langle [\![u]\!], [\![v]\!] \right\rangle_\Gamma . \tag{5}$$

Here $\{u\}$ is the average, $[\![u]\!]$ is the jump of u over the interface Γ, and α is a mesh size dependent constant.

Expanding the boundary terms in (5) leads to integrals of the form

$$\int_\Gamma u_i v_j \,\mathrm{d}\boldsymbol{A},\; i,j = 1,2. \tag{6}$$

Since the finite element meshes $\mathcal{T}_1$ and $\mathcal{T}_2$ are allowed to be non-matching, defining the interface Γ is non-trivial. In the following, we consider tetrahedral meshes $\mathcal{T}_i$, and denote the approximate surface triangulations containing the interface Γ as $\mathcal{B}_i$, $i = 1,2$.

In our application the interface has the same shape independent of the exterior domain Ω_2. To simplify the computations of the integrals we have chosen an interface Γ that lies on a spherical surface. The parametrization of the interface is non-trivial due to the singular points at the poles of the sphere, as well as the periodicity. We split the sphere into four charts and use different parametrization for each chart in order to avoid these problems. We denote the number of nodes in the triangulation $\mathcal{B}_i$ with N_i^Γ. For simplicity, we consider only one chart in the following description.

Each boundary triangle is identified with a triangle in the parametric space defined by the related chart. Hence, there exists a triangulation $\mathcal{B}_i^p$ of the parametric space corresponding to $\mathcal{B}_i$. When the two meshes are not conforming, we generate a triangular mesh $\mathcal{B}^p$ for the parametric space such that $\mathcal{B}_1^p \subset \mathcal{B}^p$ and $\mathcal{B}_2^p \subset \mathcal{B}^p$. An example of constructing the mesh $\mathcal{B}^p$ is shown in Figure 1.

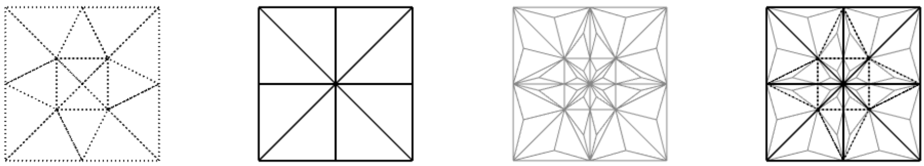

Figure 1: Two non-matching meshes $\mathcal{B}_1^p$ and $\mathcal{B}_2^p$, and a conforming mesh $\mathcal{B}^p$.

On each chart, we identify the nodes of each triangle from the mesh $\mathcal{B}_i$ with a parameter $\boldsymbol{t}_n^i$, where $n = 1, \ldots, N_i^\Gamma$. Let $K \in \mathcal{B}_i$ be a surface triangle with nodes n_1, n_2, n_3. We define a mapping $r_{i,K} : K \to K' \in \mathcal{B}^p$,

$$r_{i,K}(\boldsymbol{x}) = \sum_{j=1}^{3} \boldsymbol{t}_{n_j}^i \varphi_j(\boldsymbol{x}), \tag{7}$$

where φ_j are the linear basis functions on K. The integrals in equation (6) can now be computed as follows:

$$\int_\Gamma u_i v_j \, \mathrm{d}\boldsymbol{A} = \sum_{K \in \mathcal{B}^p} \int_K u_i(r_{i,K_i}^{-1}(\boldsymbol{x})) v_j(r_{j,K_j}^{-1}(\boldsymbol{x})) \, \mathrm{d}\boldsymbol{x}. \tag{8}$$

Here $K_i \in \mathcal{B}_i^p$ is such that $K \subset K_i$, $i = 1, 2$. The normal is chosen to be the normal of the element K.

3 RESULTS

Two different automatically extracted vocal tract geometries from MRI-sequences were used for Ω_1. To simulate the acoustic environment inside an MRI machine, a cylindrical shape with the dimensions of a head coil was used for Ω_2. Neumann boundary condition was set everywhere except on the caps of the cylinder, where Dirichlet boundary condition was used. Examples of acquired pressure distributions are shown in Figure 2. The illustrated geometries are approximately symmetrical with respect to the cutting plane. The spherical interface Γ is also shown in the visualisations.

4 CONCLUSIONS

We note that the model works well given the problem setting. When implementing this method, a lot of care is needed when it comes to handling multiple charts. However, with a fixed interface this needs to be done only once, and the convenience

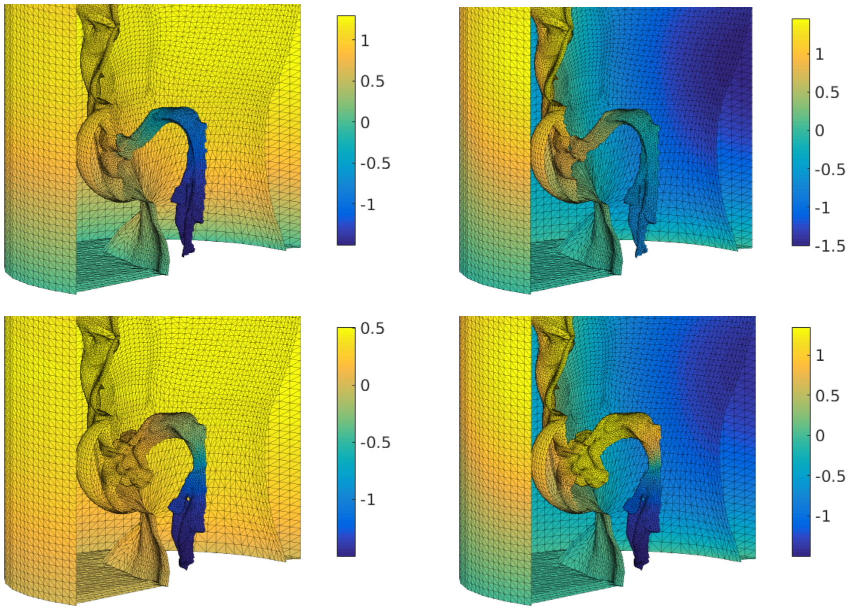

Figure 2: Scaled pressure distributions of the second and fourth modes (left/right) for two different vowels (top/bottom). We note that the pressure on the interface is nearly constant in each case.

of being able to easily exchange geometries makes up for the inital inconvenience. The next steps include setting a more complicated boundary condition at the glottis as well as incorporating a dimension reduction for the exterior acoustic space[5].

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